

Why counting counts? Supporting teachers as they support young children's mathematical learning

Early Childhood Summit
Dr. Kathy Fitzgerald
Department of Child and Family Studies
University of Tennessee
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mkfitz@utk.edu

Knowledge cannot be transmitted. It must be constructed afresh by each individual knower on the basis of what is already known and by means of strategies developed over the whole of that individual's life, both outside and inside the classroom. (Wells, 1986, p. 218)

Researchers have found:

- Successful learning of mathematics helps children create **the brain connections that support learning in other areas**, especially reading and literacy (NCR, 2009).
- Successful learning of mathematics in preschool **enables children to make use of instruction** in kindergarten and first grade.
- Without a strongly developed **intuitive foundation**, children make weak connections between concepts, create isolated, rather than interrelated, concepts and rely on underdeveloped strategies to solve problems in K and 1st grade (NCR, 2009; Griffin, 2007).

We will address two questions today:

- What overall content must teachers know to effectively support learning?
- What classroom practices should we encourage teachers to use in order to match content with how children learn?

Picture of child counting eggs

What do teachers need to know: Creating a framework for math content

- The \$10 definition of our task:
- *In our work with young children, we must support their efforts in constructing their personal*

Central Conceptual Structures

These are mental networks in which concepts are created, organized, represented and assigned meaning (Case)

What are we talking about with this language?

The **central conceptual structure** deals with the child's formation of an evolving and developing mental organization and the neural pathways needed to create and use a sense of number in meaningful and useful ways.

Central Conceptual Structure made simple

➤ Making these ideas concrete with what children need to know:

- Number names stand for quantities
- The amount represented by the number name does not change. It is stable.
- Number quantities are always in consistent relationships to other number quantities (think more or less), therefore connections between sets need to be made. **REFLECTIVE ABSTRACTION**
- **Generalizing** this idea means each successive number results in sets that contain more items and has a greater value or going in the other direction, have less objects and smaller value.

➤ Central Conceptual Structure

- This powerful organizing mechanism allows children to create, manipulate and understand these related concepts:
 - *When considering the number sequence, each number's position matters (before and after)*
 - *The 1:1 manner in which the number words are sequenced when they are assigned onto objects when counting matters*
 - *The developing understanding of the cardinal value of each number matters, i.e., when touching the third object and saying "3", the child slowly comes to know she has formed a set, a bunch, a collection containing 3 items.*

➤ So what should we do and not do

DO MORE

- Child talk
- Learning oral number names in social setting
- Counting small quantities –up to 3 or 5
- Comparing quantities and reflecting on relative size
- Using the words *more* or *less*

- Always use a variety of interesting stuff for counting
- Use counting in every day contexts
- Change tasks as children develop the concepts.

DO LESS

- Writing numerals until children ask
- Using large quantities until children ask
- Study numbers in isolation or as the number of the day.
- Waste time on meaningless tasks like counting 30 days in a month until this quantity makes sense.
- Teacher talking

Strategies of supporting this work with teachers.

- For children to own this learning, teachers must act from this fundamental belief...

A **supportive social context** is the most important factor in children's cognitive and skill development. Research has shown that...

- *Children are most likely to be advanced in a skill if they are raised in an environment that supports the development and use of the skill*
- *Children will be less advanced at a skill if they have little support or opportunity to use the skill themselves.*
- *An individual child will show various levels of cognitive functioning across various domains depending on the opportunities for developing a specific skill in the social context (Kobe)*

Therefore, what next?

- *How do we help teachers learn and appreciate the mathematics children already know when they come to school?*
- *What language would teachers use to support intuitive thinking in children?*
- *Do we have well taught and practiced routines, so teachers can interact intentionally to support learning?*

Can we agree...

- Children need a sense of control of the world around them and math ideas provide them with a means of organizing the world. Organization provides a sense of control.
- Young children have the capacity and interest to learn how math works at younger ages than we previously thought.
- Math conceptual ideas makes sense to children and they are intuitively interested in them.
- Preschool children generally do not have preconceived or negative notions of their math abilities. They like math

Additional details about the CENTRAL CONCEPTUAL STRATEGY

Translating these ideas into everyday language and classroom actions

Idea 1: Number names and sequence

- Number names matter
- The order in which we say number names matters
- Number names stand for quantities and that quantity is set and agreed upon in society. We do not have choice about what names we use for what quantity
 - There is an identified sequence or learning trajectory for most children as they learn and use number names effectively
- Social knowledge – number names are labels learned through repetition in conversation with interesting people doing interesting things.
 - They are NOT learned in structured and boring lessons or by doing WORKSHEETS
- **The learning path for most children includes:**
 - *Learning the vocabulary through chanting*
 - *Learning the vocabulary as discrete words*
 - *Counting forwards and backwards*
 - *Learning to STOP when saying the sequence of the counting words*
 - *Learning to breaking into the sequence of number words*
- **A few basic number name activities**
 - Counting forwards and backwards, counting using mental number lines –vertical actions, as well as horizontal (like up and down the ladder), circular (picture a round clock or spinner) and physical number lines
- Learning the number words as discrete words:
 - *Counting with actions using with power, softness, whispers. Only use actions that are discrete. Say NO to running five times, yes jumping five times but help children to work at controlling the jumps*
- Breaking into the counting sequence, (start to count at 3

➤ **Counting involves self-regulation**

Self-regulation is a two-part task - the act of making myself stop doing something and then start to do something else. These activities involve self-regulation if teachers engage it.

- Keeping a steady beat
- Counting in unison with others
- Not repeating number words
- Stopping when all objects have been tagged
- Tagging every object (not leaving any objects uncounted) once and only once

Learning trajectory for counting quantities

- Chanter
- Reciter
- Reciters and taggers using mostly incorrect sequence and mis-tagging
- Correct tagging and chanting for small amounts
- When asked how many, recounts everything or answers 1,2,3,4,5
- Tells last word said, but is not sure
- Becomes sure but cannot use this information to count on
- Can count on

This big idea: Numbers stand for quantities

Considerations about quantities

- Quantities are stable unless we add or take away from them (appearance does not matter)
- The size of the objects within the set does not change the quantity of items.
- There are quantities inside of quantities

Idea 2: Knowing the 1:1 manner in which the sequence is mapped onto objects when counting

The big idea: We have to count every object in the set once but only once

- To facilitate this, we have to apply mental (and /or physical organization and self-regulation onto the set)

Learning tasks involved within idea 2

- Finding things that can be counted
- Tagging one and only one item at a time for a number name (don't worry if the children initially miscall or mis-name. The other children will help them to self correct.) This learning takes time
- Tagging every item

- Tagging each item only once
- Matching finger tagging with voice tagging
- Imposing order onto the counting task
 - *Having a system for keeping track of what has been counted and what has not been counted*
 - *Talk about what is in the set and what is outside of the set*

Idea 3: When counting, children come to understand the cardinal value of each number

- The cardinal value means answering the question of HOW MANY
 - For a long time, children will count and not be able to answer the ‘how many’ question
 - This idea asks children to do two things at once -- *say the number for the last item tagged*
- AND
- *Realize that same word tells how many are in the set*

Big Ideas of Counting, in other words

- Cardinality
 - Means: Stop uttering words when each item in the collection has been identified
 - Declare that the last word uttered is the “count” of the characteristics of the objects themselves
 - Understand at the end of the process that the last word uttered establishes a characteristic of the collection rather than a characteristic of any specific object

(Van de Walle, 2009; Schwartz, 2008)

Teaching strategies

- Lots and lots of practice needed for learning to tag, to recite instead of chant, to understand how many
- For each child, focus counting quantities of a small numbers of items. under 3 or 5, until the system is really in place for that child
- Emphasize the tagging while the child is learning about the correct order of number names.
- Help child to impose order on the set
- Keep set size small while learning the tagging and collecting
- Modeling and tell thinking

Activities to support numbers as quantity

- Identifying what can be counted
 - *Mingle and count*
 - *Counting sets – how many legs on the chairs and tables –what would happen if there were less? More?*
 - *Dice games*
 - *Spinner games*
 - *How many in the cup?*
 - *How many on the spoon?*
 - *Share a favorite*

Idea 4: Understanding the generative rule which relates adjacent cardinal values

- Quantities are always in relationship to other quantities and the common step from one number to the next is plus or minus one.
- More or less -- Quantities are always more or less than other quantities
 - *The amount that a quantity varies from another yet always remains stable.*
- Idea 5: Each successive number represents a set that contains more objects and has a greater value
- 5 is more than 4 and will always be more than four no matter what is counted
- 6 comes after 5 and therefore it is a set of more objects and has a greater value
- Learning trajectory for comparative values
- No sense of the stability of set size or value
- Recognition of 3 standing for ***
- Calling a set by name
 - *That's two cars, there are 3 children*
- **Subitizing** –quick recognition of quantity in small sets without actually counting
- Developing the expectation that early numbers are smaller than later numbers which will be of greater value
- Smaller numbers combine to make larger numbers
- Teaching strategies
- Lots of practice comparing set sizes
- Lots of practice using **more, less, fewer, greater than, less than** vocabulary in context
- Practice putting sets into size (quantity) order
- Celebrate when someone “invents” the stair size connections
- Work on the idea of stability of size of set
- 3 is always *** and it is always less than 5 by 2 jumps

Learning trajectory of helping children to visualize early number relationships

- Use small numbers – compare 2 to 3 or 2 to 5 until the idea is in place
- Use spatial relationships to help children visualize quantities (dominoes and dot cards; images on playing cards)

- Play with the ideas of one (1) and two (2) more or less (advanced kids can play the sock game)

Cautions

- *Less* and *fewer* takes more work for children learn than *more*
- Children have to learn the multiple meanings of some words: “Bigger” and “larger” might refer to more quantity and not the size
- Be mindful about “cheering on” by naming the task well done; errors can often be corrected as the task is done again.

Role of language in helping children to learn

- Language is how we share our ideas
- Language is how we come to understand our own thinking
- Language is how we refine and clarify our thinking
- Thinking is very difficult, if not impossible, without language

New way of thinking

- Math requires **reflective abstraction** at all levels
- What does this phrase mean and what does it look like?
 - *Hands on manipulatives AND*
 - *Drawing and using models AND*
 - *Adding language and symbols*

Recursive process

- Recursive Process using Multiple Representations

Role of symbols

- What is a symbol?
- Preschool children are learning to become fluent with symbols in many areas
 - *Pretend play*
 - *Blocks*
 - *Stories and books*
 - *Songs*
 - In same context, numerals and numbers words are representations (stands for or reminds us of) of number ideas and rules apply

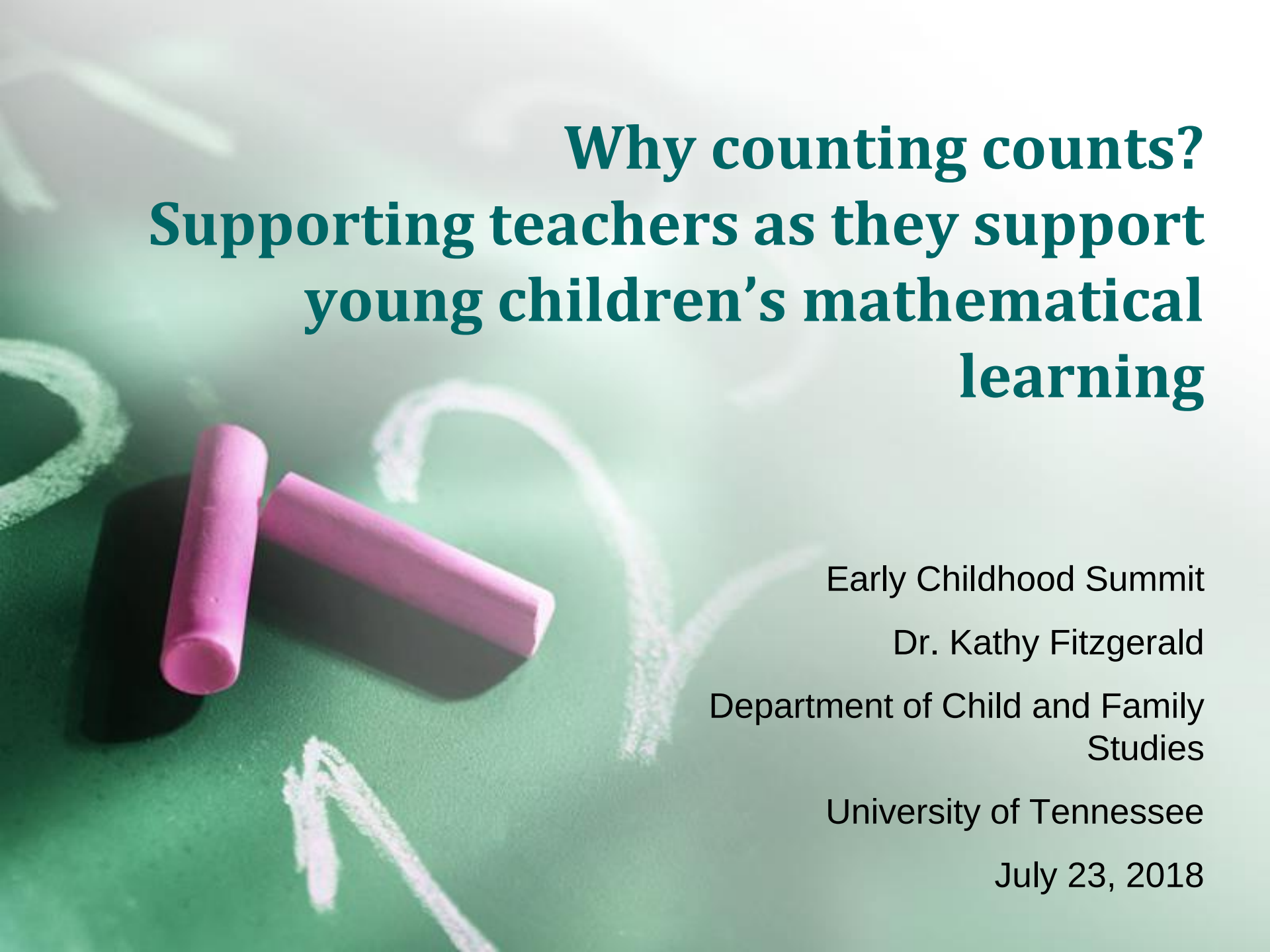
Rules about math symbols

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- Help children understand the rationale for numerals and other math symbols
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- Exercise caution when using symbols like the equal sign? What does the = sign mean?
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Do less

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A chalkboard with two pieces of pink chalk and some white chalk markings. The chalkboard is green and has a white chalk line and a white chalk arrow drawn on it. The title "Can we agree..." is written in a bold, teal font at the top left of the slide.

Can we agree...

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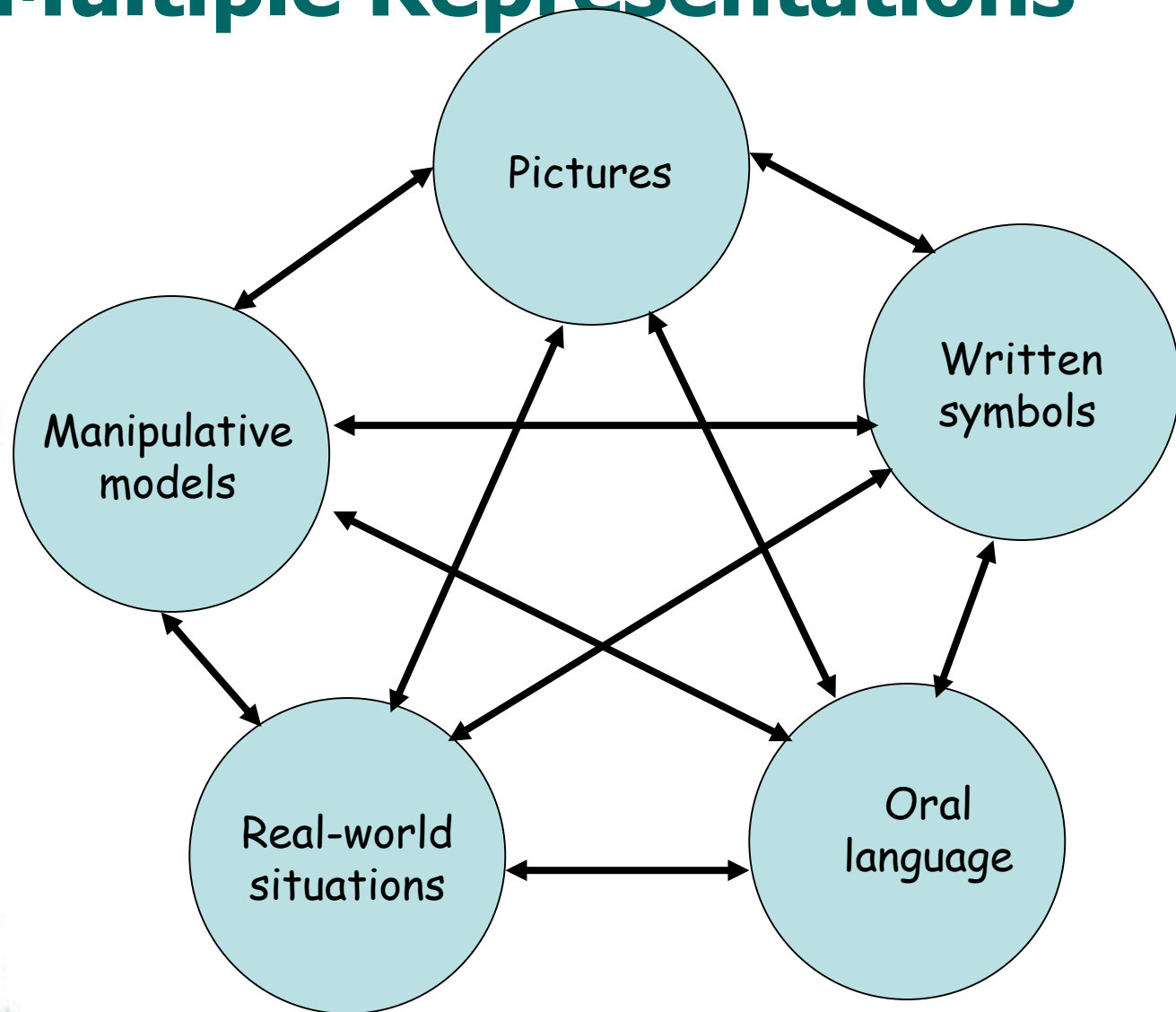
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New way of thinking

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- What does this phrase mean and what does it look like?
 - *Hands on manipulatives AND*
 - *Drawing and using models AND*
 - *Adding language and symbols*

Recursive process

Recursive Process using Multiple Representations





**See handout for more in-depth
development of these ideas**

**I can be reached at
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Translating these ideas into everyday language and classroom actions

- **Idea 1: Number names and sequence**
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Number names and sequence learning trajectory

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- They are NOT learned in structured and boring lessons or by doing WORKSHEETS
- Learning path for most children includes:
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Number name activities

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Teaching strategies

- Lots and lots of practice needed for learning to tag, to recite instead of chant, to understand how many
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- Help child to impose order on the set
- Keep set size small while learning the tagging and collecting
- Modeling and tell thinking



Activities to support numbers as quantity

- Identifying what can be counted
 - *Birthday ritual –circling the sun*
 - *Mingle and count*
 - *Counting sets – how many legs on the chairs and tables –what would happen if there were less? More?*
 - *Dice games*
 - *Spinner games*
 - *How many in the cup?*
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Idea 4: Understanding the generative rule which relates adjacent cardinal values

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Learning trajectory of helping children to visualize early number relationships

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